

$$F\left(\frac{2-\mu}{4}, 1, \mu\right) = 0, \quad (9)$$

which can be solved graphically.

The corresponding eigenfunctions in the interval  $[0, 1]$   $T(\mu R^2)$  will be orthogonal with a weight  $R(1 - R^2)$  [5], i. e., the following equality will be valid:

$$\int_0^1 R(1 - R^2) T(\mu_n R^2) T(\mu_m R^2) dR = \begin{cases} 0, & n \neq m \\ \left[ \frac{dF(a_n, 1, \mu_n)}{dR} \exp\left(-\frac{\mu_n}{2}\right) \right]^2, & n = m. \end{cases} \quad (10)$$

Following Grinberg's method [6], we will seek the solution of problem (1) with boundary conditions (2), (3), (4) in the form

$$t = \sum_{i=0}^{\infty} \frac{t_i(Z)}{\left[ \frac{dF(a_i, 1, \mu_i)}{dR} \exp\left(-\frac{\mu_i}{2}\right) \right]^2} T(\mu_i R^2), \quad (11)$$

where

$$t_i(Z) = \int_0^1 t(R, Z) R(1 - R^2) T(\mu_i R^2) dR.$$

Substituting (11) in (1) and using the boundary conditions we obtain the following ordinary linear differential equation of the first order to determine  $t_i(Z)$ :

$$\frac{dt_i}{dZ} + \mu_i^2 t_i(Z) + k_{if}(Z) = 0, \quad (12)$$

where

$$k_i = 2\mu_i \frac{dF(a_i, 1, \mu_i)}{dR} \exp\left(-\frac{\mu_i}{2}\right),$$

the solution of which will be

$$t_i = \exp(-\mu_i^2 Z) \left[ C_i - k_i \int f_i(Z) \exp(\mu_i^2 Z) dZ \right]. \quad (13)$$

The constant  $C_i$  is determined from the boundary condition (4) by multiplying this condition by the eigenfunction and integrating over the interval  $[0, 1]$ . For this we need to know the forms of func-

tions  $f(Z)$  and  $\varphi(R)$  and for the latter, as a rule, we need to assume that it can be expanded as a power series. In particular, when  $\varphi(R) \equiv 0$ ,  $f(Z) \equiv f_0 = \text{const}$

$$C_i = k_i \frac{f_0}{\mu_i^2}.$$

We denote the found values of  $C_i$  by  $C_i^0$ , and then the solution of our problem will take the form (11), where  $t_i$  is given by formula (13) with  $C_i = C_i^0$ .

The problem is solved in exactly the same way with a boundary condition of the second kind, i. e., if condition (3) of our problem is replaced by the condition

$$\frac{\partial t(1, Z)}{\partial R} = f(Z). \quad (14)$$

In this case the eigenvalues will be given by the following system:

$$\begin{aligned} 2aF(a+1, 2, \mu) &= F(a, 1, \mu), \\ a &= (2-\mu)/4. \end{aligned} \quad (15)$$

#### NOTATION

$R \equiv r/r_0$  is the dimensionless variable radius;  $Z \equiv z/r_0 \text{Pe}$  is the reduced tube length;  $\text{Pe} = 2w_{\text{av}} r_0 / \alpha$  is the Peclet number;  $r_0$  is the tube radius;  $r$  and  $z$  are cylindrical coordinates;  $w_{\text{av}}$  is the average flow velocity;  $\Gamma(a)$  is the gamma function;  $t$  is the temperature of the liquid.

#### REFERENCES

1. E. L. Ince, Ordinary Differential Equations [in Russian], GONTI, 1938.
2. N. N. Lebedev, Special Functions and Their Applications [in Russian], Gostekhizdat, 1961.
3. D. S. Kuznetsov, Special Functions [in Russian], Izd. Vysshaya shkola, Moscow, 1965.
4. V. I. Smirnov, Course of Higher Mathematics, Vols. 2 and 3, Gostekhizdat, 1964.
5. N. S. Koshlyakov, E. B. Gliner, and M. M. Smirnov, Fundamental Differential Equations of Mathematical Physics [in Russian], Fizmatgiz, 1962.
6. G. A. Grinberg, Selected Questions of Mathematical Theory of Electric and Magnetic Phenomena [in Russian], Izd. AN SSSR, 1948.

2 July 1966

Kalinin Polytechnic Institute,  
Leningrad

#### THERMAL CONDUCTIVITY OF SOME METALS AND ALLOYS IN THE TEMPERATURE RANGE 4.2-273° K

B. A. Merisov, V. I. Khotkevich, G. M. Zlobintsev, and V. V. Kozinets

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 675-677, 1967

UDC 536.512

In the thermal potentiometer method of measurement of thermal conductivity of metals (Fig. 1) [1], the total heater power may be represented in the form

$$W = W_1 + W_2,$$

where  $W_1$  is the power passing through the cross section of the specimen  $T_1$ ; and  $W_2$  is the power scattered by radiation from the specimen surface  $S_1$ , located below the section  $T_1$ .

Similarly

$$W_1 = W_3 + W_4,$$

where  $W_4$  is the power passing through the specimen cross section  $T_2$ ; and  $W_3$  is the power scattered by radiation from the specimen surface located between sections  $T_1$  and  $T_2$ .

For  $T_1 - T_0 \approx 1^\circ$ ,  $W_3$  is not less than 1% of  $W$ , right up to 100-150° K.  $W_4$  is a quantity of the second order of smallness in compari-

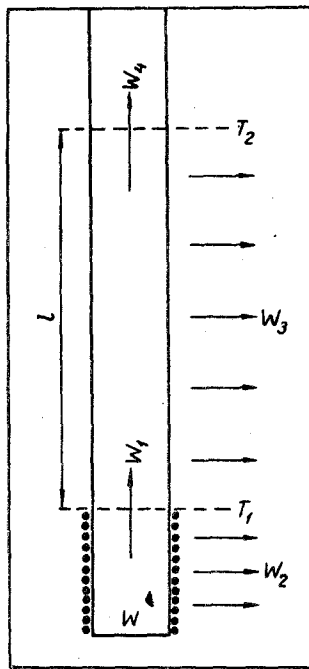


Fig. 1. Schematic of the power balance in the thermal potentiometer method of thermal conductivity measurement.

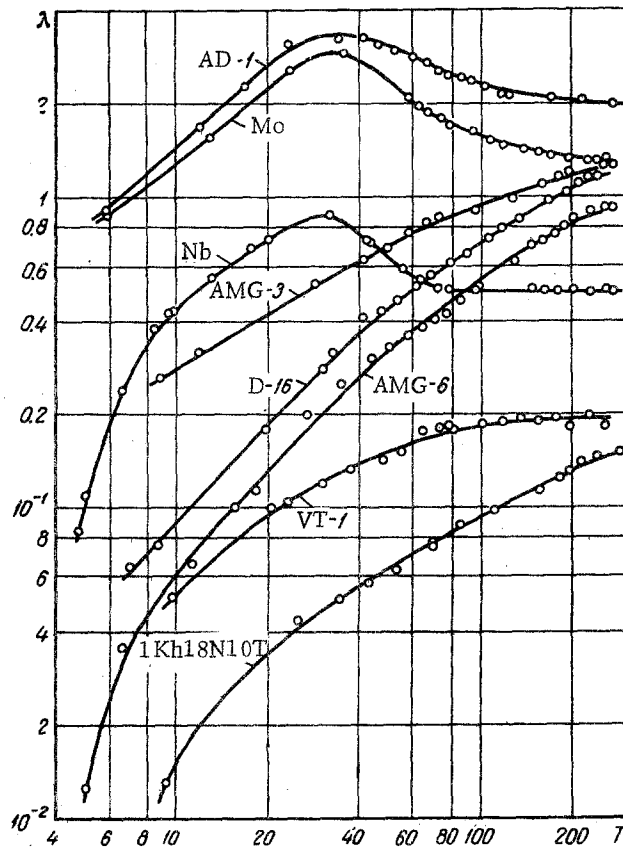


Fig. 2. Temperature dependence of the thermal conductivities  $\lambda$ ,  $W/cm \cdot deg$ , of several metals and several alloys.

son with  $W_2$ . Under these conditions the Fourier formula for the thermal conductivity is valid:

$$\lambda = Wl/\Delta TS, \quad (1)$$

where

$$\Delta T = T_1 - T_2.$$

At higher temperatures it is necessary to take into account the quantity  $W_2$ ; according to the Stefan-Boltzmann equation,

$$W_2 = \sigma \epsilon S_1 (T_1^4 - T_0^4), \quad (2)$$

where  $T_0$  is the screen temperature.

Neglecting  $W_3$  as before, we obtain, for the thermal conductivity,

$$\lambda = \frac{W_1 l}{\Delta TS} = \frac{Wl}{\Delta TS} - \frac{\sigma \epsilon S_1 (T_1^4 - T_0^4) l}{\Delta TS}. \quad (3)$$

The second term in this equation is a correction taking into account the power dissipated in radiation. Since  $\epsilon$  is usually unknown, we need to determine the correction experimentally. Taking into account the fact that  $\sigma$  is constant, that  $S_1$ ,  $S$  and  $l$  do not change during the experiment, and that the total emissivity of metals increases linearly with temperature as a first approximation ( $\epsilon = \epsilon_0 T$ ) [2], formula (3) may be rewritten as

$$\lambda = \lambda' - \frac{AT_1(T_1^4 - T_0^4)}{\Delta T}, \quad (4)$$

where

$$\lambda' = \frac{Wl}{\Delta TS}; \quad A = \frac{\sigma \epsilon_0 S_1 l}{S}.$$

If we determine the values of  $\lambda'_1$  and  $\lambda'_2$  at two different screen temperatures  $T_{01}$  and  $T_{02}$  and a single specimen heater temperature  $T_1$ , then, solving the two equations (which were obtained from (4) by substitution of  $T_{01}$ ,  $\lambda'_1$  and  $T_{02}$ ,  $\lambda'_2$ ) with the two unknowns ( $\lambda$  and  $A$ ), we find

$$A = \frac{(\lambda'_1 - \lambda'_2) \Delta T_1 \Delta T_2}{T_1 [\Delta T_2 (T_1^4 - T_{01}^4) - \Delta T_1 (T_1^4 - T_{02}^4)]}. \quad (5)$$

Because we can no longer neglect  $W_3$  as the temperature increases, we may expect to find an upper temperature limit for application of formula (4).

We conducted experiments to determine this limit, on good and bad heat conductors—copper and 1Kh18N10T steel. A deviation of the experimental data, as computed from formula (4), from the tabulated values began for copper from a temperature of 270° K, and for steel from 245° K. Of course, the upper temperature limit of application of the present method depends also on the constructional peculiarities of the equipment. Thus, a decrease in the temperature gradient along the specimen and of the gap between the specimen and the screen reduces the value of  $W_3$ , which shifts the limit toward higher temperatures.

To measure the thermal conductivity of metals we constructed an equipment similar to that described earlier [1]. As a temperature sensor in the range 4.2° to 20° K we used a TSG-2 germanium resistance thermometer. From 20° to 273° K we used a differential copper-

constantan thermopile. Intermediate temperatures were achieved and maintained by means of the thermal regulator described in [3].

Measurement of the thermal conductivity involved measurement of the heater power and of the temperatures  $T_1$ ,  $T_2$ , and  $T_0$  under steady thermal conditions. From these data the quantity  $\lambda'$  was calculated for each temperature point, determined as  $(T_1 + T_2)/2$ .

Determination of  $A$  (formula (4)) was carried out as follows. The quantity  $\lambda' = \lambda'_1$  was measured with a cryostat immersed in liquid nitrogen, and the temperature  $T_1$  maintained close to 273° K. Then the nitrogen bath was replaced with an ice bath; then the temperature  $T_1$  also became close to 273° K. The value of  $\lambda'$  measured under these conditions was different from  $\lambda'_1$  and will be denoted by  $\lambda'_2$ . The screen temperature and the temperature gradients along the specimen were different for these cases and will be denoted by  $T_{01}$  and  $T_{02}$ , and  $\Delta T_1$  and  $\Delta T_2$ , respectively. The information thus obtained was enough for calculating  $A$  from formula (5). From the calculated value of  $A$  and the experimentally determined  $\lambda'$ , the thermal conductivity  $\lambda$  was determined for each temperature point, from formula (4).

The advantages of the above method in comparison with other existing methods consist, in our opinion, of the following:

1. The absence of a supplementary heater for the screen. The presence of such a heater, although it reduces the noncalculated power flux going into radiation, substantially complicates the electrical measurement system and increases the time required for thermal equilibrium.

2. The method does not require special treatment of the specimen surface to reduce the total emissivity (which lowers the scattered power), nor, on the other hand, to increase it (which allows the specimen to be regarded as an absolutely black body).

3. The method does not require supplementary experiment over the whole temperature range in order to calculate the radiation.

In the present work measurements were made of thermal conductivity of technically pure niobium and molybdenum, as well as of the alloys AD-1, AMG-6, AMG-3, D-16, VT-1, and 1Kh18N10T. The results of the measurements are shown in Fig. 2. The error in determining the thermal conductivity did not exceed 5% over the whole temperature range.

#### NOTATION

$W$  is the power;  $T$  is the absolute temperature;  $S$  is the area of the transverse cross section of the specimen;  $S_1$  is the partial surface area of the specimen;  $l$  is the length;  $\lambda$  is the thermal conductivity;  $\epsilon$  is the total emissivity of the surface  $S_1$ ;  $\sigma$  is the Stefan-Boltzmann constant.

#### REFERENCES

1. G. K. White, *Experimental Methods in Low Temperature Physics* [Russian translation], Fizmatgiz, 1961.
2. Collection: "Methods of temperature measurement," Part 2 [Russian translation], IL, 1954.
3. N. G. Alexseev, V. A. Prokhorov, and K. V. Chmutov, *Electronic Instruments and Systems in Physical and Chemical Research* [in Russian], Goskhimizdat, 1961.